Automated Cost Estimating Integrated Tools

Everything you wanted to know about Learning with Rate (but were afraid to ask)

Tecolote Research, Inc.

Abstract

Most cost analysts are familiar with learning curves and many use ACE's basic learning capabilities in their estimates. Many still have questions about ACE's advanced learning capabilities, such as Rate adjusted and broken learning, and when to use them.

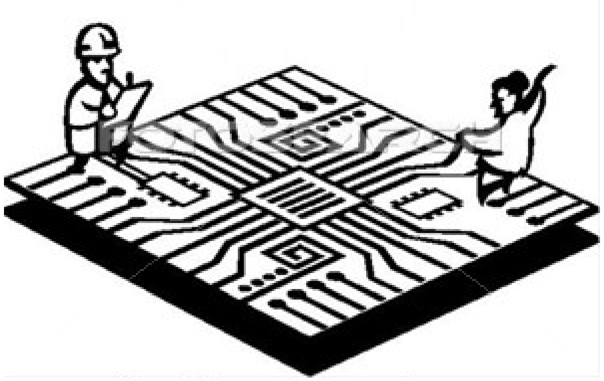
This presentation takes a practical look at learning curves. It discusses what learning curves mean, where they come from, and explains when and how to use adjusted or modified methods (e.g., broken learning, rate adjusted learning). When does using a rate term make sense based on what is going on in a manufacturing process and how do you calculate it? What scenarios exist where broken learning should be used? Come to this presentation to learn the answers to these questions and more.

Outline

- **What is Learning?**
  - What do learning curves mean?
  - Where do they come from?
- **Learning with Rate**
- **Other Rate Curve Considerations**

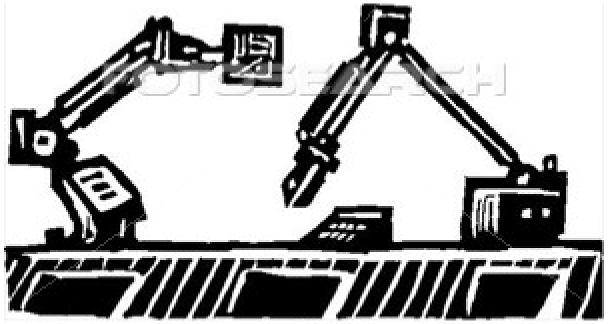
Introduction: What is Learning?

Production Start



Early Production

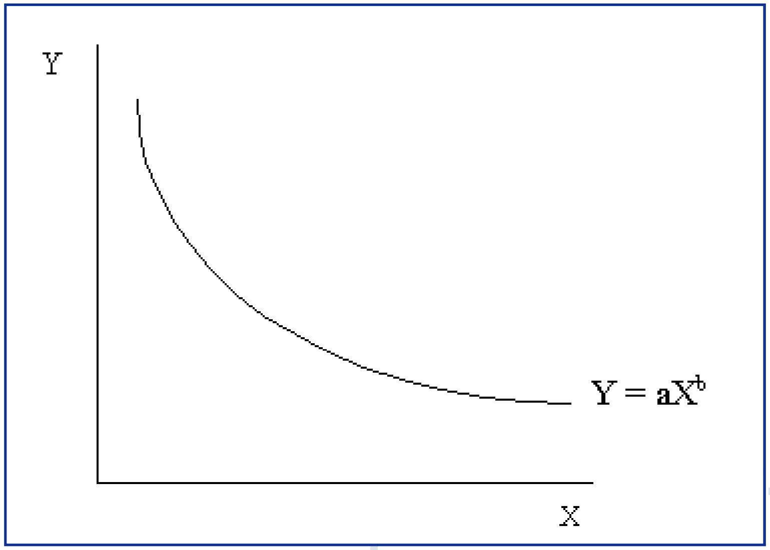
Peak Efficiency



- **A learning curve is a mathematical model (equation) of productivity improvement in a manufacturing process.**

- **Introduced in 1936 by T.P. Wright**
  - Based on production of airplane assemblies

- **Standard Model: The direct labor man-hours necessary to complete a unit of production will decrease by a constant percentage each time the production quantity is doubled**



Unit vs. Cum Avg Theory

- **Two common theories of learning**
  - Unit - As the total quantity of units doubles, the cost per **unit** goes down by a constant percentage
  - Cum Average - As the total quantity of units doubles, the **average cost for x units** goes down by a constant percentage

?

- **$Y = A * x^b$,** where

  Y = Unit or Cum Average Cost

  A = First Unit Cost (T_1)

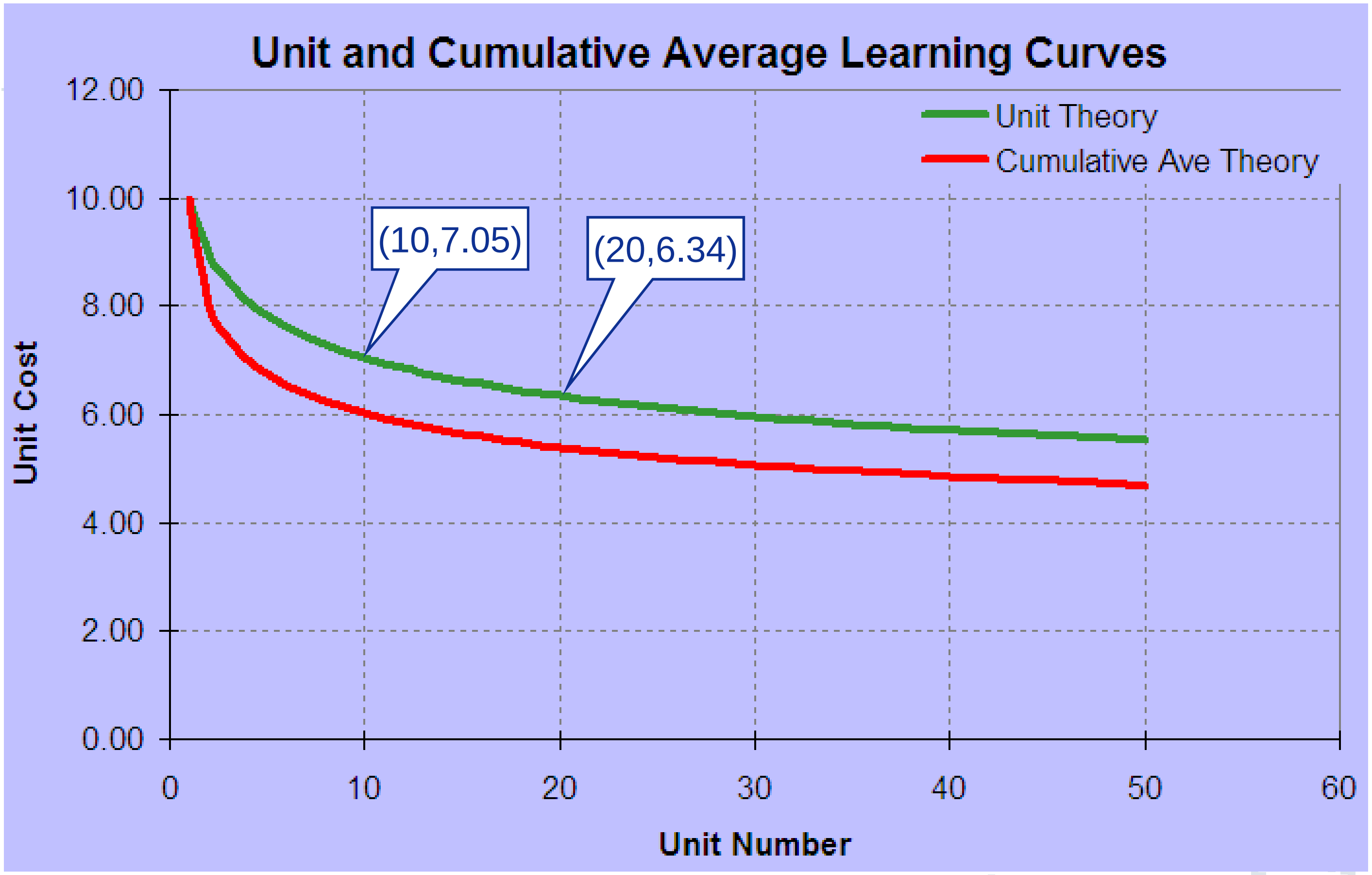
  x = Unit Number

  b = Learning Curve Exponent = ln(Slope)/ln(2)

- **Example:** A 5% unit cost decrease corresponds to a 95% slope

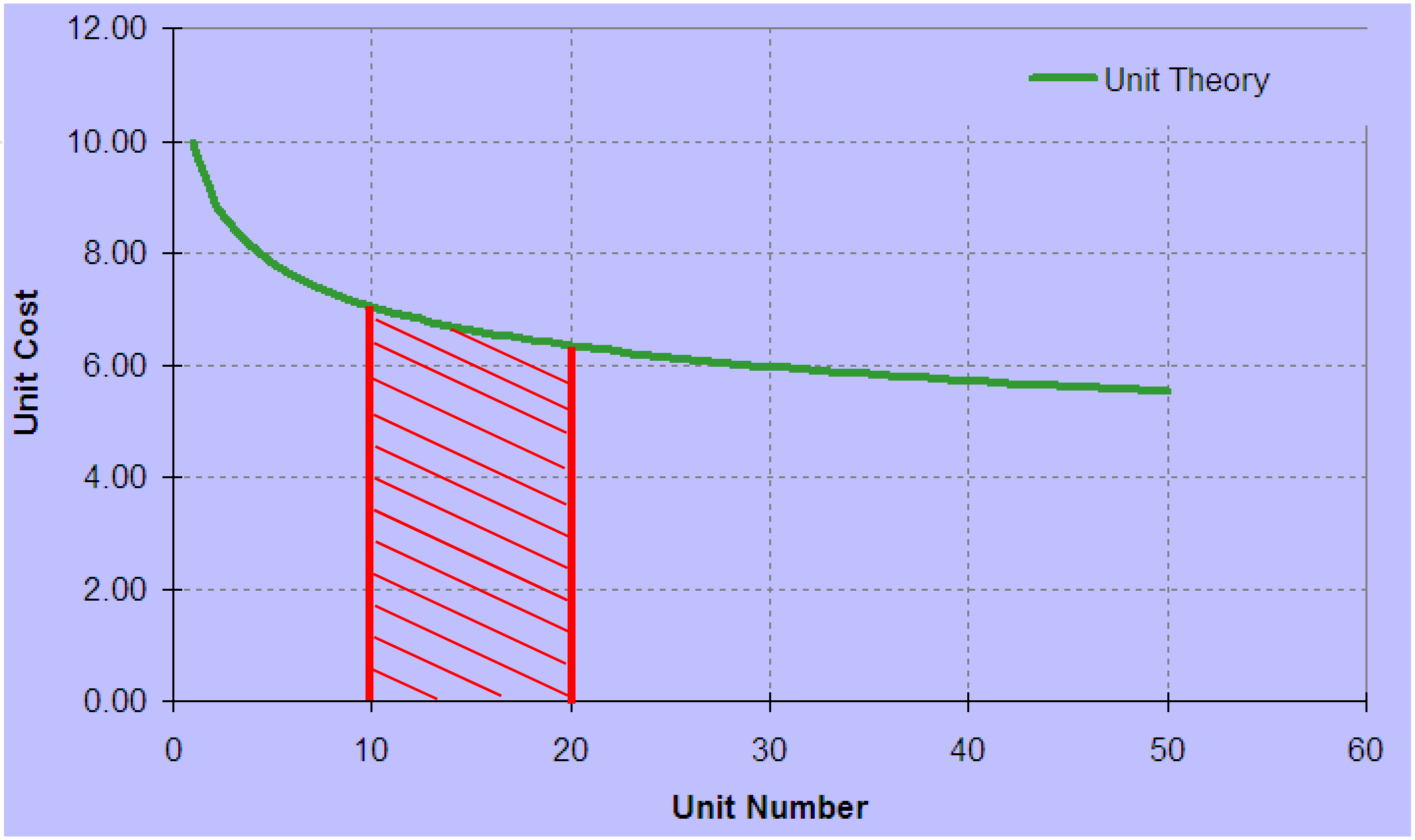
Unit vs. Cum Avg Theory

- **Example 90% unit curve: doubling the unit number leads to a 10% decrease in the unit cost**



Unit Curves & Lot Costs

- **Unit Cost: $C_i = T_1 * (i) ^ b$**
- **Lot Cost of units 10 thru 20:**
  - $C_{10} + C_{11} + \ldots C_{20}$
  - Approximated by area under the curve for large lots



Where And Why?

- **Regression model from historical data**
  - Historical cost reporting (e.g., CSDR, CPR) for sequential production lots
    - Cost reporting by contract
    - CSDR Report Types
      - 1921: recurring, non-recurring, and total dollars
      - 1921-1: hours and $ visibility by functional category (e.g., manufacturing)
      - 1921-2: Progress Curve; unit-by-unit or lot-by-lot detail
  - Contractors may have more detailed or “unit” data
- **Typical Applications**
  - Use earlier lot data to project future lots for a system
  - Use learning curve from analogous system to estimate new system
  - More advanced: pooled learning regression

Cost Reports: 1921

SECURITY CLASSIFICATION **Unclassified**

COST DATA SUMMARY REPORT

Form Approved OMB No. 0704-0188

The public reporting burden for this collection of information is estimated to average 8 hours per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing the burden, to Department of Defense, Washington Headquarters Services, Executive Services Directorate (0704-0188). Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to any penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number.

PLEASE DO NOT RETURN YOUR COMPLETED FORM TO THE ABOVE ORGANIZATION.

1. PROGRAM
a. MDAP:
b. PHASE:

2. PRIME MISSION PRODUCT

3. CONTRACTOR TYPE (X one)
☐ PRIME / ASSOCIATE
☐ DIRECT-REPORTING SUBCONTRACTOR

4. NAME/ADDRESS (Include ZIP Code)

5. APPROVED PLAN NUMBER

6. CUSTOMER (DIRECT-REPORTING SUBCONTRACTOR USE ONLY)

7. CONTRACT

8. CONTRACT PRI

9. CONTRACT

10. TYPE ACTION
a. CONTRACT NO.:
b. LATEST MODIFICATION:
c. SOLICITATION NO.:
d. NAME:

11. PERIOD OF PERFORMANCE
a. START DATE *(YYYYMMDD)*:
b. END DATE *(YYYYMMDD)*:

12. APPROPRIATION
☐ RDT&E
☐ PROCUREMENT
☐ O&M

13. REPORT CYCLE
☐ INITIAL
☐ INTERIM
☐ FINAL

14. SUBMISSION NUMBER

15. RESUBMISSION NUI

16. REPORT AS OF *(YYYYMMDD)*

17. NAME (Last, First, Middle Initial)

18. DEPARTMENT

19. TELEPHONE NUMBER (Include Area Code)

20. EMAIL ADDRESS

21. DATE PREPARED *(YYYYMMDD)*

WBS ELEMENT CODE	WBS REPORTING ELEMENTS	NUMBER OF UNITS TO DATE	COSTS INCURRED TO DATE			NUMBER OF UNITS AT COMPLETION	COSTS INCURRED AT COMPLETION		
			NONRECURRING	RECURRING	TOTAL		NONRECURRING	RECURRING	TOTAL
A	B	C	D	E	F	G	H	I	J

Page 1

Cost Reports: 1921-1

FUNCTIONAL COST-HOUR REPORT

Form Approved OMB No. 0704-0188

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4. NAME/ADDRESS (Include Zip Code)

5. APPROVED PLAN NUMBER

6. CUSTOMER (Direct-Reporting Subcontractor Use Only)

7. TYPE ACTION a. CONTRACT NO.: b. LATEST MODIFICATION: c. SOLICITATION NO.: d. NAME:

8. PERIOD OF PERFORMANCE a. START DATE *(YYYYMMDD)*: b. END DATE *(YYYYMMDD)*:

9. REPORT CYCLE ☐ INITIAL ☐ INTERIM ☐ FINAL

10. SUBMISSION NUMBER

11. RESUBMISSION NUMBER

12. REPORT AS OF (YYYYMMDD

13. NAME (Last, First, Middle Initial)

14. DEPARTMENT

15. TELEPHONE NO. (Include Area Code)

16. EMAIL ADDRESS

17. DATE PREPARED (YYYYMMI

18. WBS ELEMENT CODE

19. WBS REPORTING ELEMENT

20. NUMBER OF UNITS a. TO DATE: b. AT COMPLETION:

21. APPROPRIATION ☐ RDT&E ☐ PROCUREMENT ☐ O&M

FUNCTIONAL DATA ELEMENTS	COSTS AND HOURS INCURRED TO DATE			COSTS AND HOURS INCURRED AT COMPLETION		
	A. NONRECURRING	B. RECURRING	C. TOTAL	D. NONRECURRING	E. RECURRING	F.TOTAL
ENGINEERING						
(1) DIRECT ENGINEERING LABOR HOURS						
(2) DIRECT ENGINEERING LABOR DOLLARS						
(3) ENGINEERING OVERHEAD DOLLARS						
(4) TOTAL ENGINEERING DOLLARS						
MANUFACTURING OPERATIONS						
(5) DIRECT TOOLING LABOR HOURS						
(6) DIRECT TOOLING LABOR DOLLARS						
(7) DIRECT TOOLING & EQUIPMENT DOLLARS						
(8) DIRECT QUALITY CONTROL LABOR HOURS						
(9) DIRECT QUALITY CONTROL LABOR DOLLARS						
(10) DIRECT MANUFACTURING LABOR HOURS						
(11) DIRECT MANUFACTURING LABOR DOLLARS						
(12) MANUFACTURING OPERATIONS OVERHEAD DOLLARS *(Including Tooling and Quality Control*						
(13) TOTAL MANUFACTURING OPERATIONS DOLLARS *(Sum of rows 6, 7, 9, 11, and 12)*						
MATERIALS						
(14) RAW MATERIAL DOLLARS						
(15) PURCHASED PARTS DOLLARS						
(16) PURCHASED EQUIPMENT DOLLARS						
(17) MATERIAL HANDLING/OVERHEAD DOLLARS						
(18) TOTAL DIRECT-REPORTING SUBCONTRACTOR DOLLARS						
(19) TOTAL MATERIAL DOLLARS						
OTHER COSTS						
(20) OTHER COSTS NOT SHOWN ELSEWHERE *(Specify in Remarks)*						
SUMMARY						
(21) TOTAL COST *(Direct and Overhead)*						

Page 1

Cost Reports: 1921-2

PROGRESS CURVE REPORT

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4. NAME/ADDRESS (Include ZIP Code)

5. APPROVED PLAN NUMBER

6. CUSTOMER (Direct-Reporting Subcontractor Use On

7. TYPE ACTION
a. CONTRACT NO.:
b. LATEST MODIFICATION:
b. SOLICITATION NO.:
d. NAME:

8. PERIOD OF PERFORMANCE
a. START DATE *(YYYYMMDD)*:
b. END DATE *(YYYYMMDD)*:

9. REPORT CYCLE
☐ INITIAL
☐ INTERIM
☐ FINAL

10. SUBMISSION NUMBER

11. RESUBMISSION NUMBER

12. REPORT AS OF (YYYYMMDD)

13. NAME (Last, First, Middle Initial)

14. DEPARTMENT

15. TELEPHONE NO. (Include Area Code)

16. E-MAIL ADDRESS

17. DATE PREPARED (YYYYMMDD)

18. WBS ELEMENT CODE

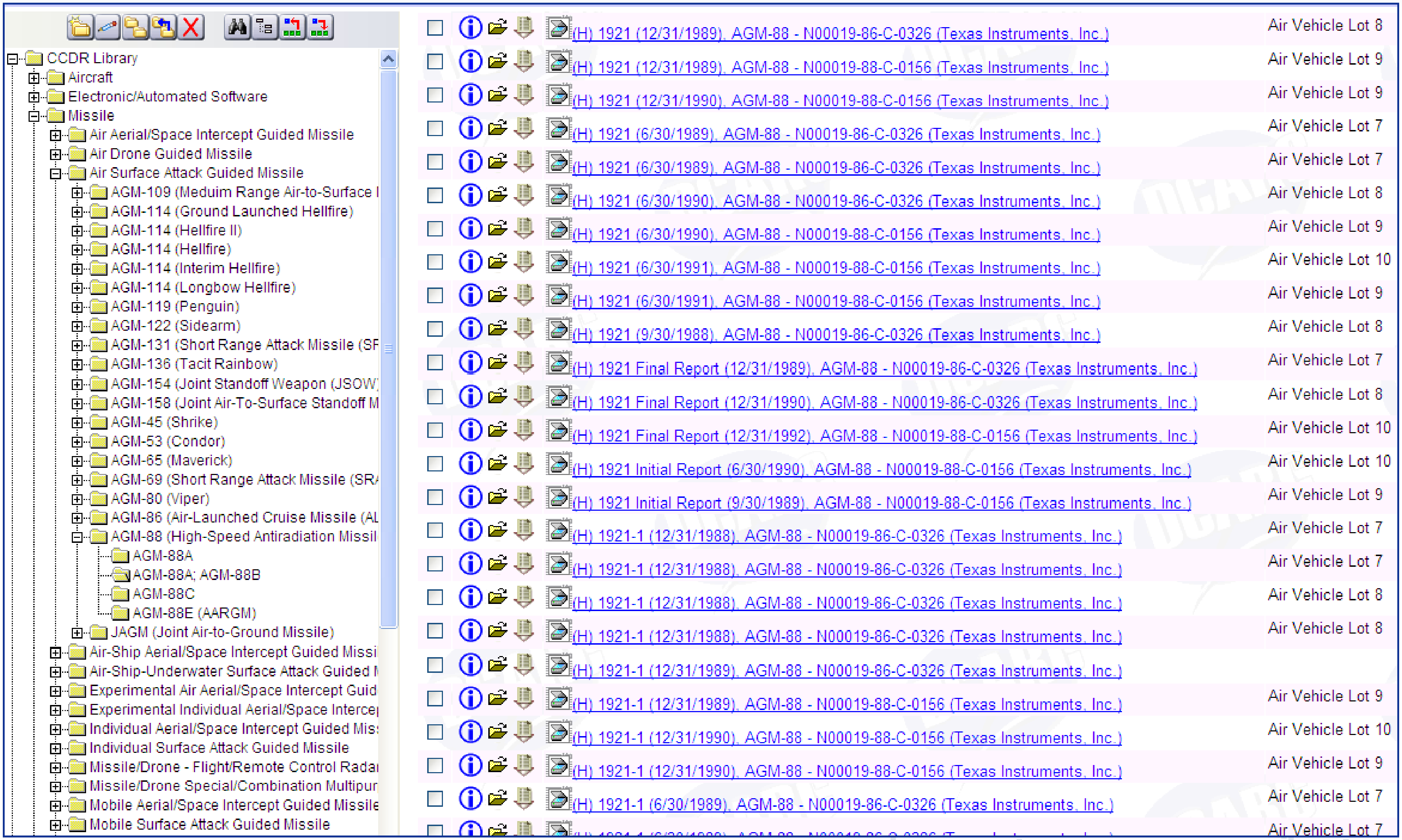
19. WBS REPORTING ELEMENT

20. UNITS/LOTS COMPLETED
☐ UNIT TOTAL
☐ LOT TOTAL

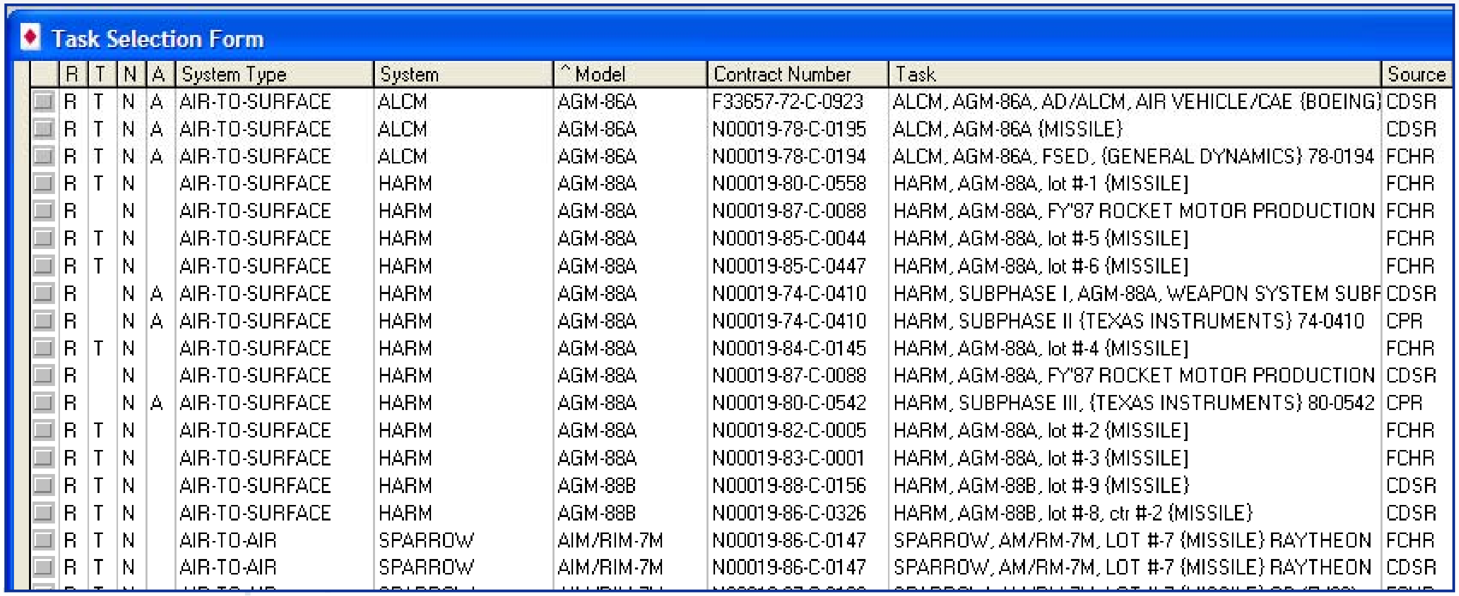
21. APPROPRIATION
☐ RDT&E
☐ PROCUREMENT
☐ O&M

DATA ELEMENTS	A. COMPLETED UNITS/LOTS				B. WORK IN PROCESS (WIP)	C. TOTAL DIRECT COSTS AND HOURS INCURRED TO DATE
	A1	A2	A3	A4		
(1) MODEL AND SERIES						
(2) FIRST UNIT						
(3) LAST UNIT						
(4) CONCURRENT UNITS						
CHARACTERISTICS						
(5a) Weight						
(5b) Speed						
(5c) Power						
ENGINEERING (RECURRING ONLY)						
(6) DIRECT ENGINEERING LABOR HOURS						
(7) DIRECT ENGINEERING LABOR DOLLARS						
MANUFACTURING OPERATIONS (RECURRING ONLY)						
(8) DIRECT TOOLING LABOR HOURS						
(9) DIRECT TOOLING LABOR DOLLARS						
(10) DIRECT TOOLING & EQUIPMENT DOLLARS						
(11) DIRECT QUALITY CONTROL LABOR HOURS						
(12) DIRECT QUALITY CONTROL LABOR DOLLARS						
(13) DIRECT MANUFACTURING LABOR HOURS						
(14) DIRECT MANUFACTURING LABOR DOLLARS						
(15) TOTAL DIRECT MANUFACTURING OPERATIONS DOLLARS *(Sum of rows 9, 10, 12, and 14)*						
MATERIALS (RECURRING ONLY)						
(16) RAW MATERIALS DOLLARS						
(17) PURCHASED PARTS DOLLARS						
(18) PURCHASED EQUIPMENT DOLLARS						
(19) TOTAL DIRECT-REPORTING SUBCONTRACTOR DOLLARS						
(20) TOTAL DIRECT MATERIAL DOLLARS						
OTHER COSTS (RECURRING ONLY)						
(21) OTHER DIRECT COSTS NOT SHOWN ELSEWHERE *(Specify in Remarks)*						
SUMMARY (RECURRING ONLY)						
(22) TOTAL DIRECT COST						

Data Sources: DACIMS (DCARC)

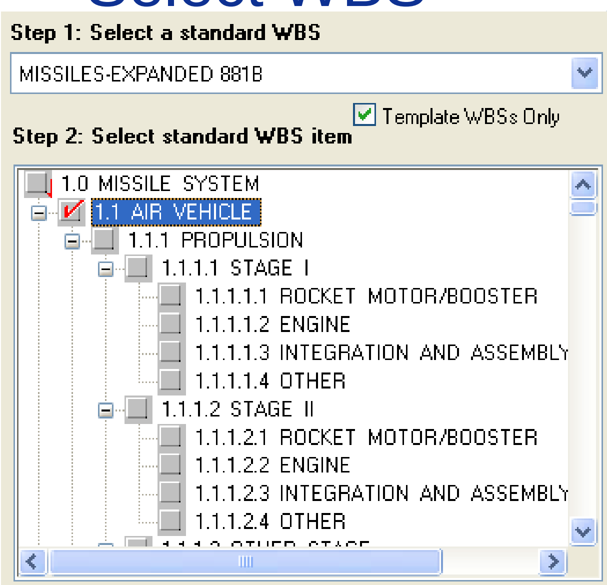


Data Sources: ACDB (ODASA-CE)

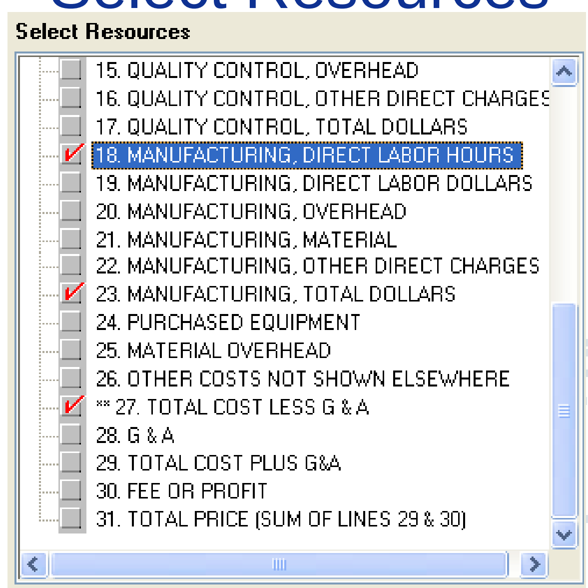

Task Selection Form

	R	T	N	A	System Type	System	^ Model	Contract Number	Task	Source
	R	T	N	A	AIR-TO-SURFACE	ALCM	AGM-86A	F33657-72-C-0923	ALCM, AGM-86A, AD/ALCM, AIR VEHICLE/CAE (BOEING)	CDSR
	R	T	N	A	AIR-TO-SURFACE	ALCM	AGM-86A	N00019-78-C-0195	ALCM, AGM-86A (MISSILE)	CDSR
	R	T	N	A	AIR-TO-SURFACE	ALCM	AGM-86A	N00019-78-C-0194	ALCM, AGM-86A, FSED, (GENERAL DYNAMICS) 78-0194	FCHR
	R	T	N		AIR-TO-SURFACE	HARM	AGM-88A	N00019-80-C-0558	HARM, AGM-88A, lot #-1 (MISSILE)	FCHR
	R		N		AIR-TO-SURFACE	HARM	AGM-88A	N00019-87-C-0088	HARM, AGM-88A, FY'87 ROCKET MOTOR PRODUCTION	FCHR
	R	T	N		AIR-TO-SURFACE	HARM	AGM-88A	N00019-85-C-0044	HARM, AGM-88A, lot #-5 (MISSILE)	FCHR
	R	T	N		AIR-TO-SURFACE	HARM	AGM-88A	N00019-85-C-0447	HARM, AGM-88A, lot #-6 (MISSILE)	FCHR
	R		N	A	AIR-TO-SURFACE	HARM	AGM-88A	N00019-74-C-0410	HARM, SUBPHASE I, AGM-88A, WEAPON SYSTEM SUBF	CDSR
	R		N	A	AIR-TO-SURFACE	HARM	AGM-88A	N00019-74-C-0410	HARM, SUBPHASE II (TEXAS INSTRUMENTS) 74-0410	CPR
	R	T	N		AIR-TO-SURFACE	HARM	AGM-88A	N00019-84-C-0145	HARM, AGM-88A, lot #-4 (MISSILE)	FCHR
	R		N		AIR-TO-SURFACE	HARM	AGM-88A	N00019-87-C-0088	HARM, AGM-88A, FY'87 ROCKET MOTOR PRODUCTION	CDSR
	R		N	A	AIR-TO-SURFACE	HARM	AGM-88A	N00019-80-C-0542	HARM, SUBPHASE III, (TEXAS INSTRUMENTS) 80-0542	CPR
	R	T	N		AIR-TO-SURFACE	HARM	AGM-88A	N00019-82-C-0005	HARM, AGM-88A, lot #-2 (MISSILE)	FCHR
	R	T	N		AIR-TO-SURFACE	HARM	AGM-88A	N00019-83-C-0001	HARM, AGM-88A, lot #-3 (MISSILE)	FCHR
	R	T	N		AIR-TO-SURFACE	HARM	AGM-88B	N00019-88-C-0156	HARM, AGM-88B, lot #-9 (MISSILE)	CDSR
	R	T	N		AIR-TO-SURFACE	HARM	AGM-88B	N00019-86-C-0326	HARM, AGM-88B, lot #-8, ctr #-2 (MISSILE)	CDSR
	R	T	N		AIR-TO-AIR	SPARROW	AIM/RIM-7M	N00019-86-C-0147	SPARROW, AM/RM-7M, LOT #-7 (MISSILE) RAYTHEON	FCHR
	R	T	N		AIR-TO-AIR	SPARROW	AIM/RIM-7M	N00019-86-C-0147	SPARROW, AM/RM-7M, LOT #-7 (MISSILE) RAYTHEON	CDSR

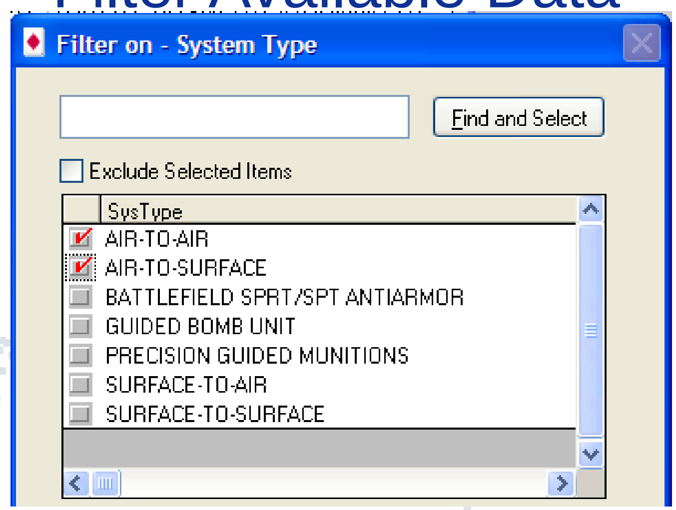
Select WBS



Select Resources



Filter Available Data

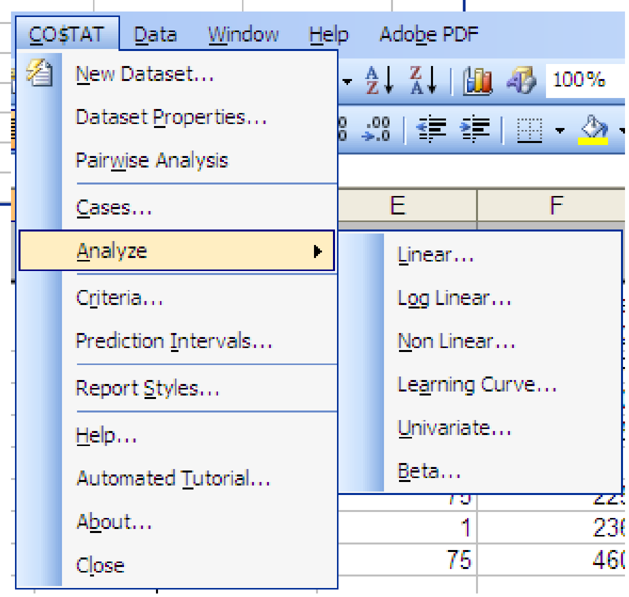


Learning Curves in CO$TAT

Variables	Variable ID	TASK1	TASK2	TASK3	TASK4	TASK5	TASK6	TASK7	TA
Task		Johns Missile Lot 1	Johns Missile Lot 1	Johns Missile Lot 2	Johns Missile Lot 2	Johns Missile Lot 3	Johns Missile Lot 4	Johns Missile Lot 5	Johns
$ Unit, HR Unit (Infl Index: 09/10/2004)		$K, KHR	$K, KHR	$K, KHR	$K, KHR	$K, KHR	$K, KHR	$K, KHR	$
Base Year		2000	2000	2000	2000	2000	2000	2000	
Total % Spent (ACWP/LRE)		95.15	94	53.65	99.32	55.03	93.64	82.83	
System Buy Quantity	QTY	107	75	225	200	200	534	372	
System First Unit	FU	129	1	236	76	76	461	276	
System Last Unit	LU	235	75	460	275	275	994	647	
Unit Cost	UC	1202	822	644	613	504	469	526	
1.1 AIR VEHICLE									
R,27. TOTAL COST LESS G & A									
2.0 PRODUCTION	RecCst	128647	61685	144821	122563	100784	250504	195519	
TOT,27. TOTAL COST LESS G & A 2.0 PRODUCTION		239795	72822	193834	145876	119741	314377	249415	
TECHNICAL DATA FOR 1.1 AIR VEHICLE									
DIAMETER (in)									
LENGTH (in)									
RANGE - EFFECTIVE RANGE (km) <HIGH>									
RANGE - MSL FLYOUT (MAX) (mi)									
SPAN - MAX (in)									
VELOCITY - MAX (ft/s)									
WEIGHT (lb)									
YEAR IN SERVICE									

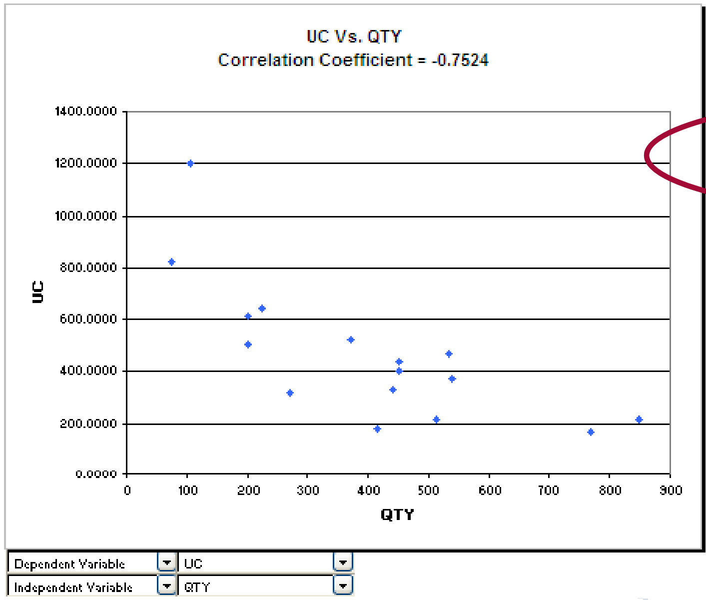
Sample Data

What would explain this?



Learning Curves in CO$TAT

II. Scatter Plot



Learning Analysis for Dataset Missile AV, UnitCurve

Sunday, January 18, 10:12 pm

I. Model Form and Equation Table

Model Form:	Unweighted Learning Curve (Unit Theory)
Number of Observations Used:	16
Equation in Unit Space:	UNIT_COST = 3659 * UNIT_NUM ^ (-0.3345)
T1:	3659.4126
Quantity Slope:	79.31%
Dependent Variable:	RecCst
Quantity Variable:	First Unit and Last Unit Variables

II. Fit Measures (in Fit Space)

Coefficient Statistics Summary

Variable	Coefficient	Std Dev of Coef	Beta Value	T-Statistic (Coef/SD)	Prob Not Zero
Intercept	8.2051	0.3426		23.9470	1.0000
CUM_QTY	-0.3345	0.0509	-0.8692	-6.5775	1.0000

Goodness-of-Fit Statistics

Std Error (SE)	R-Squared	R-Squared (Adj)	Pearson's Corr Coef
0.2808	75.55%	73.81%	0.8692

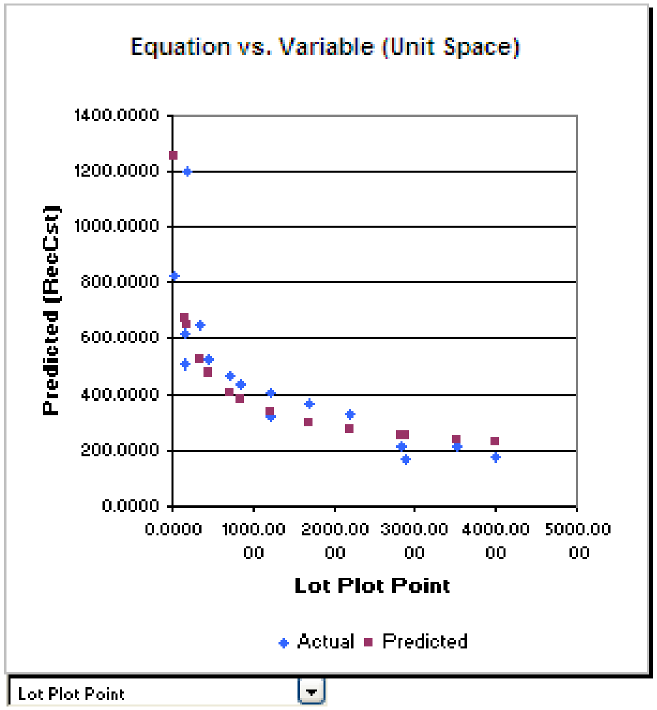
Summary of Predictive Measures

Average Actual (Avg Act)	464.3039
Standard Error (SE)	202.5707
Root Mean Square (RMS) of % Errors	26.58%
Mean Absolute Deviation (Mad) of % Errors	22.21%
Coef of Variation based on Std Error (SE/Avg Act)	43.63%
Coef of Variation based on MAD Res (MAD Res/Avg Act)	25.50%
Pearson's Correlation Coefficient between Act & Pred	72.90%
Adjusted R-Squared in Unit Space	42.69%

Learning Curves in CO$TAT

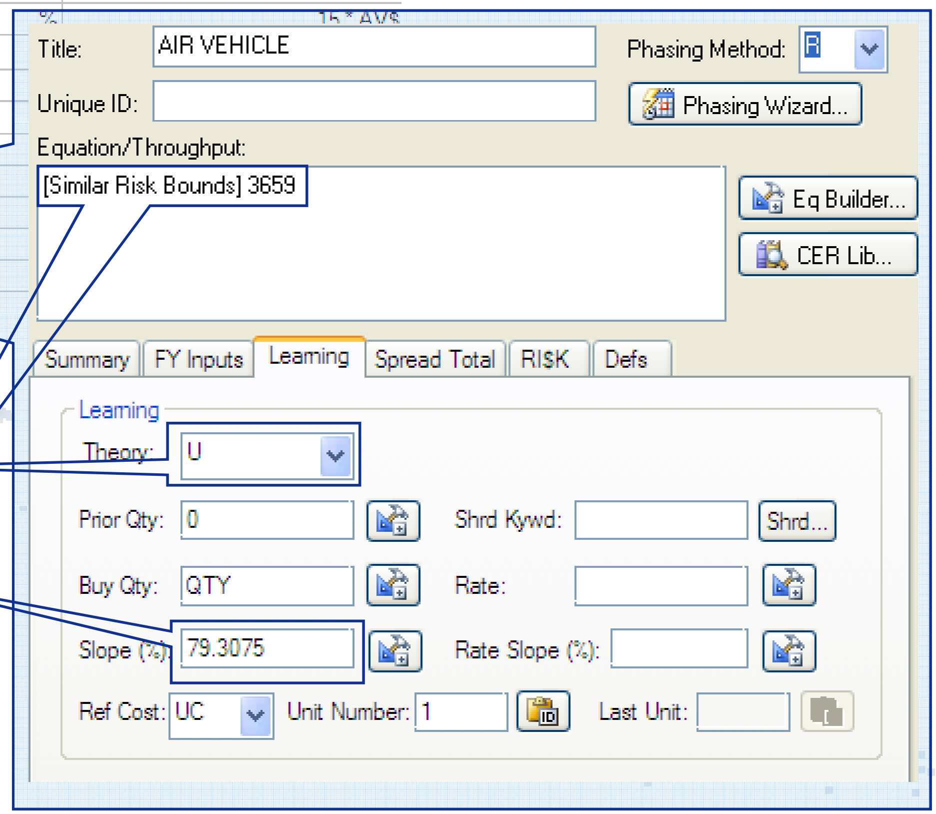
V. Learning Curve Data & Predicted LAC

Lot #	First Unit #	Last Unit #	Lot Plot Point	LOT_QTY	Actual LAC	Predicted LAC
1	129.0000	235.0000	178.4265	107.0000	1202.3067	646.1959
2	1.0000	75.0000	24.4845	75.0000	822.4667	1255.6479
3	236.0000	460.0000	339.6967	225.0000	643.6500	520.9973
4	76.0000	275.0000	161.6399	200.0000	612.8170	667.9079
5	76.0000	275.0000	161.6399	200.0000	503.9210	667.9079
6	461.0000	994.0000	704.9480	534.0000	469.1093	408.1185
7	276.0000	647.0000	444.1177	372.0000	525.5888	476.3222
8	995.0000	1444.0000	1210.1901	450.0000	405.0014	340.6334
9	648.0000	1097.0000	859.3814	450.0000	436.8199	381.9550
10	1445.0000	1984.0000	1704.9859	540.0000	371.9586	303.7354
11	1098.0000	1367.0000	1229.2017	270.0000	321.4575	338.8621
12	1985.0000	2425.0000	2200.0840	441.0000	331.7447	278.9096
13	2426.0000	3274.0000	2835.8613	849.0000	214.2241	
14	3275.0000	3787.0000	3526.8506	513.0000	217.4960	
15	2522.0000	3290.0000	2894.6369	769.0000	171.2030	
16	3788.0000	4201.0000	3992.1126	414.0000	179.0970	



Learning Curves in ACE

WBS/CES Description	Approp	Unique ID	BASELINE	Phasing Method	Equation / Throughput
ARMY CES (MISSILE)			$ 244.038 *		
RDT&E FUNDED ELEMENTS	RDTEA		$ 107.067 *		
DEVELOPMENT ENGINEERING	RDTEA	DE$	$ 41.941 *		
AIR VEHICLE (Hardware)	RDTEA		$ 20.626 *	BE	HW_LRATE$ * HW_SM
AIR VEHICLE (Software)	RDTEA		$ 21.315 *	BE	SW_LRATE$ * SW_SM
PROTOTYPE MANUFACTURING	RDTEA	PROTO$	$ 25.515 *		
AIR VEHICLE	RDTEA	AV$	$ 22.187 *	%	
Component 1	RDTEA		$ 1.962 *		COMP1$ * ProtoQ
Component 2	RDTEA		$ 17.675 *		COMP2$ * ProtoQ2
COTS Component	RDTEA		$ 2.550 *		COMP3$ * ProtoQ
INTEG/ASSY/TEST/CHKOUT	RDTEA		$ 3.328 *	%	15 * AV$
SYSTEMS ENGINEERING/MGMT	RDTEA		$ 25.499 *		
OTHER RDT&E	RDTEA		$ 14.112 *		
PROCUREMENT FUNDED ELEMEN	MIPA		$ 94.234 *		
RECURRING PRODUCTION	MIPA		$ 90.911 *		
MANUFACTURING	MIPA		$ 85.590 *		
AIR VEHICLE	MIPA		$ 85.590 *		
INTEG/ASSY/TEST/CHKOUT	MIPA		$ 0.000 *		
RECURRING ENGINEERING	MIPA		$ 5.320 *		



CO$TAT

Model Form:	Unweighted Learning Curve (Unit Theory)
Number of Observations Used:	16
Equation in Unit Space:	UNIT_COST = 3659 * UNIT_NUM ^ (-0.3345)
T1:	3659.4126
Quantity Slope:	79.31%
Dependent Variable:	RecCst
Quantity Variable:	First Unit and Last Unit Variables

Learning with Rate

Rate-Adjusted Learning

- **Used to capture variations in production rate**
  - Unit costs may decrease more quickly than expected during build-up of production line
  - Unit costs may slightly decrease during peak efficiency
  - Unit costs may increase during ramp-down as number of units being built per year decreases
- **Commonly used for long production lines (many years, high total quantities)**
  - Typical for Aircraft and Missiles
  - Rate may dominate learning with vehicles
  - Others?

Rate-Adjusted Learning

- Typical Model: $\mathbf{Y = A * x^{b} * r^{c}}$, where

  Y = Unit or Cum Average Cost

  A = First Unit Cost (assumed rate of 1)

  x = Unit Number

  b = Learning Curve Exponent = ln(Slope)/ln(2)

  r = Annual Production Rate

  c = Rate Learning Exponent = ln(Rate Slope)/ln(2)

- **What does "assumed rate of 1" mean?**
  - Set unit number (x) and rate (r) to 1.
  - Yields Y=A so the first unit cost A is based on r=1.
- **What other options are there for the rate term (r)?**
  - Why pick annual production rate?
  - Do we actually use "annual production rate"?

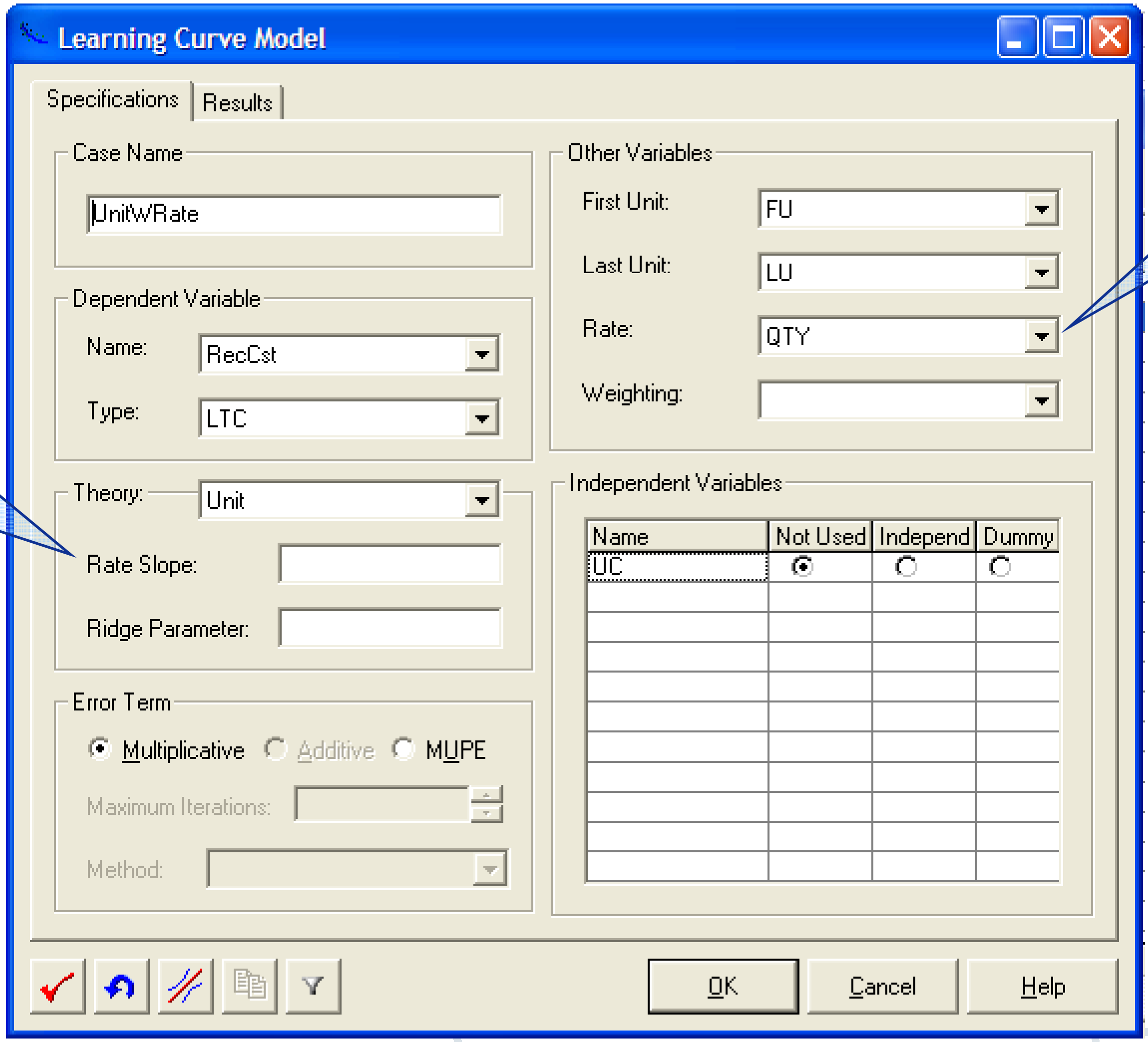
Rate-Adjusted Learning

- **Difficult to fit a Rate Curve with just the early lots**
  - Early in production cycle, learning term dominates

- **What if you just have data on the first couple productions lots and you want to generate a learning curve to estimate the future lots?**
  - Assume a fixed rate slope?
  - Fit a rate-adjusted curve to data for analogous system (adjust heuristically to the data for your system)
  - Pooled Regression

- **What if you just have data for 1 lot and want to assume learning or learning with rate (in ACE)?**

Learning with Rate in CO$TAT



Assume a Rate Slope

Fitting a rate curve

Learning with Rate in CO$TAT

I. Model Form and Equation Table

Model Form:	Unweighted Learning Curve (Unit Theory)
Number of Observations Used:	10
Equation in Unit Space:	UNIT_COST = 5964 * UNIT_NUM ^ (-0.2713) * QTY ^ (-0.1555)
T1:	5964.3377
Quantity Slope:	82.86%
Rate Slope:	89.78%
Dependent Variable:	RecCst
Quantity Variable:	First Unit and Last Unit Variables

II. Fit Measures (in Fit Space)

Coefficient Statistics Summary

Variable	Coefficient	Std Dev of Coef	Beta Value	T-Statistic (Coef/SD)	Prob Not Zero
Intercept	8.6936	0.7995		10.8732	1.0000
CUM_QTY	-0.2713	0.1114	-0.7020	-2.4348	0.9700
QTY	-0.1555	0.2355	-0.1904	-0.6604	0.4795

*** Low T ***

Goodness-of-Fit Statistics

Std Error (SE)	R-Squared	R-Squared (Adj)	Pearson's Corr Coef
0.2853	76.55%	72.95%	0.8750

Multicollinearity Analysis

Indep Variables	Indiv R-Sqr (%)	F-Stats	Prob Related to Other Vars	Indiv R-Sqr/Model R-Sqr	Flags
CUM_QTY	78.30%	50.5216	1.0000	1.0228	X
QTY	78.30%	50.5216	1.0000	1.0228	X

X = The indicated independent variable could be harmfully correlated to the other independent variables, i.e., it has a nearly better fit using the remaining independent variables than the dependent variable.

Multicollinearity is a common problem. Why?

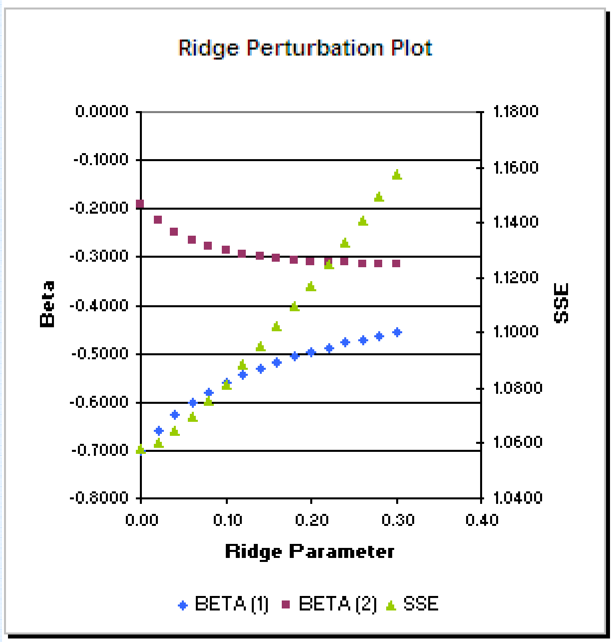
Learning with Rate in CO$TAT

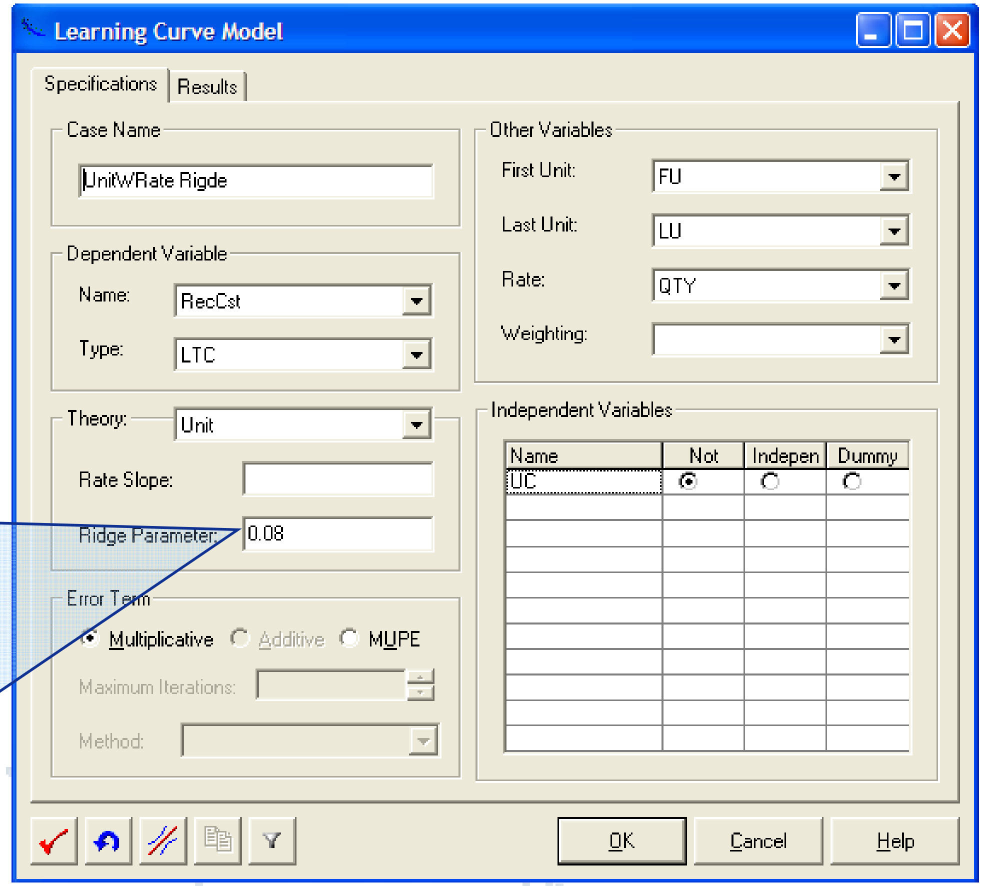
The Art of Ridge Regression

VII. Ridge Pertubation Parameter (RPP) & Related Statistics

RPP by Non-iterative Procedure	0.0682
RPP by Iterative Procedure	0.0854
Von Neumann Test for Autocorrelation	2.0065
Durbin-Watson Statistic	1.8811
Determinate of (X'X)	0.2170
Measure of Ill conditioning	4.6087

VIII. Charts





Learning with Rate in ACE

Without Ridge Parameter

Model Form:	Unweighted Learning Curve (Unit Theory)
Number of Observations Used:	16
Equation in Unit Space:	UNIT_COST = 5964 * UNIT_NUM ^ (-0.2713) * QTY ^ (-0.1555)
T1:	5964.3377
Quantity Slope:	82.86%
Rate Slope:	89.78%
Dependent Variable:	RecCst
Quantity Variable:	First Unit and Last Unit Variables

With Ridge Parameter

I. Model Form and Equation Table

Model Form:	Unweighted Learning Curve (Unit Theory)
Number of Observations Used:	16
Equation in Unit Space:	UNIT_COST = 6579 * UNIT_NUM ^ (-0.2237) * QTY ^ (-0.2263)
T1:	6579.3258
Quantity Slope:	85.64%
Rate Slope:	85.48%
Ridge Parameter Specified:	0.0800
Dependent Variable:	RecCst
Quantity Variable:	First Unit and Last Unit Variables

II. Fit Measures (in Fit Space)

Coefficient Statistics Summary

Variable	Coefficient	Std Dev of Coef	Beta Value	T-Statistic (Coef/SD)	Prob Not Zero
Intercept	8.7917	0.8060		10.9084	1.0000
CUM_QTY	-0.2237	0.0688	-0.5789	-3.2532	0.9937
QTY	-0.2263	0.1453	-0.2771	-1.5572	0.8567

Ridge Regression "Fixes" Significance of Rate Term

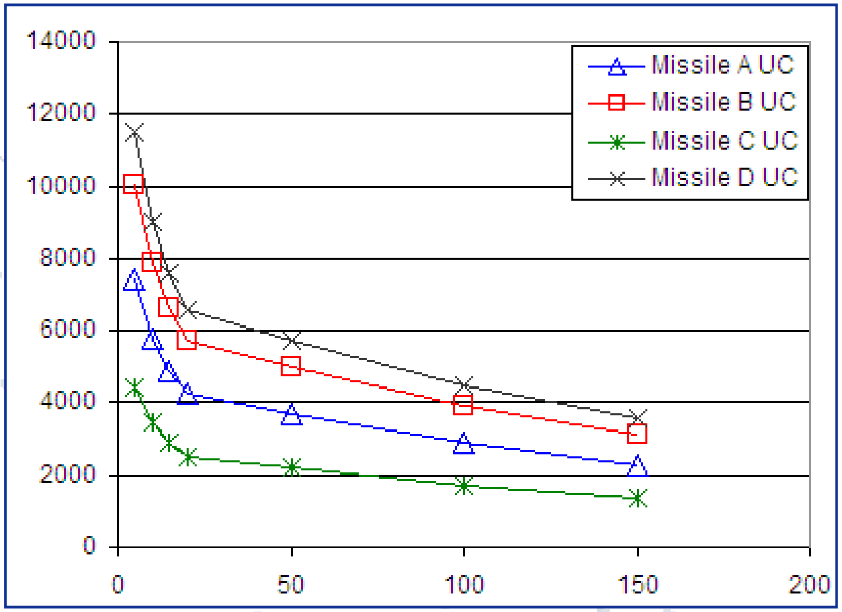
Summary of Predictive Measures

Average Actual (Avg Act)	464.3039
Standard Error (SE)	188.9897
Root Mean Square (RMS) of % Errors	26.80%
Mean Absolute Deviation (Mad) of % Errors	23.47%
Coef of Variation based on Std Error (SE/Avg Act)	40.70%
Coef of Variation based on MAD Res (MAD Res/Avg Act)	24.99%
Pearson's Correlation Coefficient between Act & Pred	77.33%
Adjusted R-Squared in Unit Space	50.11%

Other Rate Curve Considerations

Pooled Regression

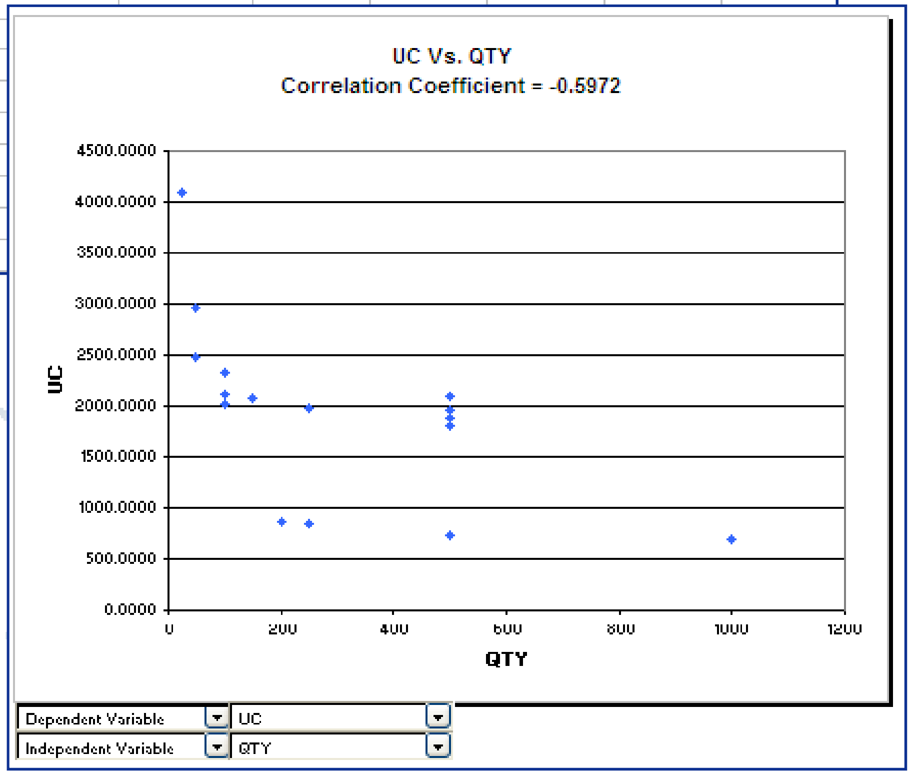
- **Consider developing a composite learning curve across multiple systems.**
  - Include one or more production lots from different systems.
  - Introduce one or more technical characteristics to stratify data.



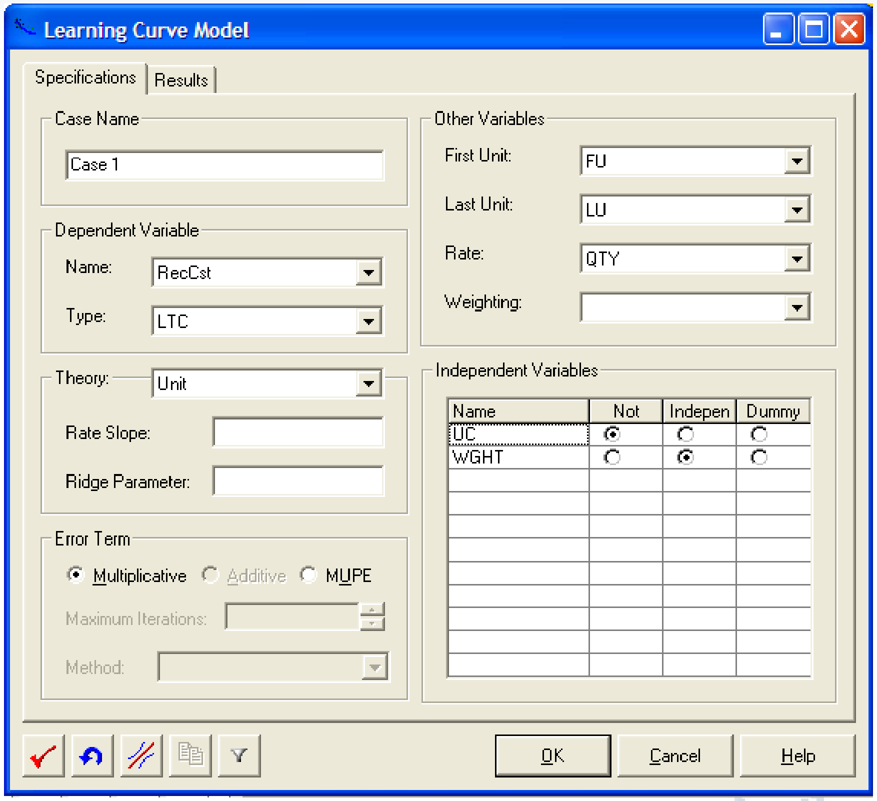
- **Example: UC = (T1 * WGHT ^{b1}) * Unit# ^{b2} * Rate ^{b3}**

Pooled Regression In CO$TAT

Variables	Variable ID	TASK1	TASK2	TASK3	TASK4	TASK5	TASK6	TASK7	TASK8	TASK9	TASK10
Task		Msl A Lot 1	Msl A Lot 2	Msl A Lot 3	Msl A Lot 4	Msl A Lot 5	Msl B Lot 3	Msl C Lot 2	Msl C Lot 3	Msl C Lot 4	Msl D Lot 3
$ Unit, HR Unit (Infl Index: 09/10/2004)		$K, KHR	$K, KHR	$K, KHR	$K, KHR	$K, KHR	$K, KHR	$K, KHR	$K, KHR	$K, KHR	$K, KHR
Base Year		2000	2000	2000	2000	2000	2000	2000	2000	2000	2000
Total % Spent (ACWP/LRE)		95.15	94	53.65	99.32	55.03	93.64	82.83	69.7	95.61	77.65
System Buy Quantity	QTY	25	50	100	100	100	1000	200	250	500	500
System First Unit	FU	1	26	76	176	276	15524	850	1050	1300	650
System Last Unit	LU	25	75	175	275	375	16523	1049	1299	1799	1149
Unit Cost	UC	4087	2962	2324	2109	2014	701	862	852	734	2101
1.1 AIR VEHICLE	RecCst	102174	148120	232442	210949	201421	701417	172478	213107	367128	1050686
TECHNICAL DATA FOR 1.1 AIR VEHICLE											
DIAMETER (in)											
LENGTH (in)											
RANGE - EFFECTIVE RANGE (km) <HIGH>											
RANGE - MSL FLYOUT (MAX) (mi)											
SPAN - MAX (in)											
VELOCITY - MAX (ft/s)											
WEIGHT (lb)	WGHT	250	250	250							
YEAR IN SERVICE											



Pooled Regression In CO$TAT



I. Model Form and Equation Table

Model Form:	Unweighted Learning Curve (Unit Theory)
Number of Observations Used:	16
Equation in Unit Space:	UNIT_COST = 167.3 * UNIT_NUM ^ (-0.167) * QTY ^ (-0.1689) * WGHT ^ 0.7641
T1:	167.3170
Quantity Slope:	89.07%
Rate Slope:	88.95%
Dependent Variable:	RecCst
Quantity Variable:	First Unit and Last Unit Variables

II. Fit Measures (in Fit Space)

Coefficient Statistics Summary

Variable	Coefficient	Std Dev of Coef	Beta Value	T-Statistic (Coef/SD)	Prob Not Zero
Intercept	5.1199	0.3417		14.9853	1.0000
CUM_QTY	-0.1670	0.0238	-0.5962	-7.0078	1.0000
QTY	-0.1689	0.0400	-0.3455	-4.2263	0.9988
WGHT	0.7641	0.0587	0.7341	13.0072	1.0000

Goodness-of-Fit Statistics

Std Error (SE)	R-Squared	R-Squared (Adj)	Pearson's Corr Coef
0.1056	96.62%	95.77%	0.9829

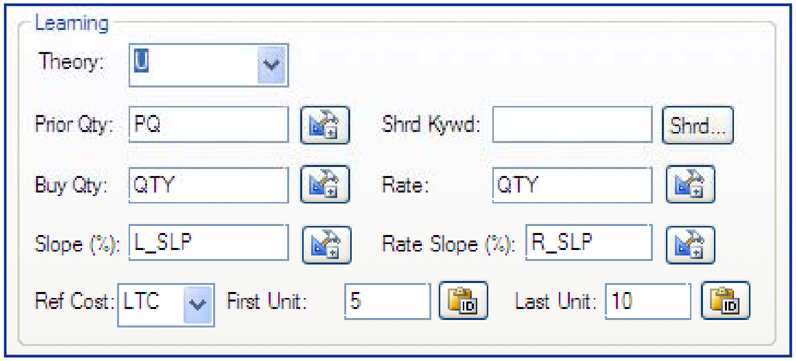
Summary of Predictive Measures

Average Actual (Avg Act)	1933.1814
Standard Error (SE)	168.4335
Root Mean Square (RMS) of % Errors	10.32%
Mean Absolute Deviation (Mad) of % Errors	6.20%
Coef of Variation based on Std Error (SE/Avg Act)	8.71%
Coef of Variation based on MAD Res (MAD Res/Avg Act)	5.24%
Pearson's Correlation Coefficient between Act & Pred	98.93%
Adjusted R-Squared in Unit Space	96.31%

Real-World What-If

- **Consider: What if you have actual cost reporting for a single (early) production lot.**
  - Lot Total Cost of Units 5 through 10 is $1000 (say LRIP 1).
  - You need to use this information to estimate future LRIP and productions.
  - You want to assume (for lack of better information) a 90% unit theory learning curve with 95% rate.

WBS/CES Description	Unique ID	Point Estimate	Phasing Method	Equation / Throughput	Start Date	Finish Date
Calculated LTC Units 5 - 10		875.823 *	R	LTCi		
Prior QTY	PQ	4.000 *	C	4		
Buy QTY	QTY	6.000 *	F	6	2009	2009
Input LTC (Units 5-10)	LTCi	1,000.000 *	C	1000		
Learning Slope	L_SLP	90.000 *	C	90		
Rate Slope	R_SLP	95.000 *	C	95		



- **What is wrong with this picture?**

- **A reference cost on a rate adjusted learning curve has an assumed rate of 1.**
  - The given LTC = $1000 is for units 5 – 10; annual rate is 6 units
  - Need to manually adjust reference cost to rate of 1.
  - Key fact: The rate exponent = **ln(Rate Slope)/ln(2)**

WBS/CES Description	Unique ID	Point Estimate	Phasing Method	Equation / Throughput	Start Date	Finish Date
Calculated LTC Units 5 - 10		1,000.000 *	R	LTCi / (6 ^ (ln(R_SLP/100) / ln(2)))		
Calculated LTC Units 5 - 10 Wrong		875.823 *	R	LTCi		
Prior QTY	PQ	4.000 *	C	4		
Buy QTY	QTY	6.000 *	F	6	2009	2009
Input LTC (Units 5-10)	LTCi	1,000.000 *	C	1000		
Learning Slope	L_SLP	90.000 *	C	90		
Rate Slope	R_SLP	95.000 *	C	95		

Summary

- Learning curves are commonly used in many cost modeling activities
  - Rate adjusted learning is frequently needed to model long production runs and the later production lots.
  - To correctly understand and apply learning, you need to consider the source… historical data.
- ACEIT makes using learning easy
  - ACDB & CO$TAT: access historical data and developing curves
  - ACE: Easy implementation of learning and learning with rate